Selkirk College

SCHOOL OF UNIVERSITY ARTS AND SCIENCES

MATH 100 – REVIEW PACKAGE

Gearing up for calculus and preparing for the Assessment Test that everybody writes on __________ at _______.

You are strongly encouraged not to use a calculator for any of the following questions, since calculators will not be allowed during the Assessment Test. The test is closed book.

A. ARITHMETIC SKILLS

Example 1: Simplify each

a. $\dfrac{-(-3)^3+(-5)}{-16-5\cdot 3}$

Solution:

$$\frac{-(-3)^3+(-5)}{-16-5\cdot 3}=\frac{27-5}{-16-15}$$
$$=-\frac{22}{31}$$

b. $\dfrac{5}{4}\div\dfrac{3}{2}-\left(-\dfrac{1}{3}-\dfrac{2}{9}\right)$

Solution:

$$\frac{5}{4}\div\frac{3}{2}-\left(-\frac{1}{3}-\frac{2}{9}\right)=\frac{5}{4}\times\frac{2}{3}+\frac{1}{3}+\frac{2}{9}$$
$$=\frac{5}{6}+\frac{3}{9}+\frac{2}{9}$$
$$=\frac{5}{6}+\frac{5}{9}$$
$$=\frac{15}{18}+\frac{10}{18}$$
$$=\frac{25}{18}$$

c. $\dfrac{3}{\sqrt{12}}$

Solution:

$$\frac{3}{\sqrt{12}}=\frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{3\sqrt{3}}{2\cdot 3}$$
$$=\frac{\sqrt{3}}{2}$$

1. Simplify:

a. $\dfrac{4+(-3)^2}{6-5\cdot 3}$ b. $\dfrac{4\times\dfrac{1}{16}}{\dfrac{1}{3}-\dfrac{1}{2}}$ c. $\dfrac{\sqrt{175}}{5}$

Note: All answers are given at the end of this package.

B. ALGEBRA

Example 2: Simplify each expression

a. $\dfrac{6-3k}{k^2-4}$

<u>*Solution*</u>

$$\frac{6-3k}{k^2-4}=\frac{3(2-k)}{(k+2)(k-2)}$$
$$=\frac{-3}{k+2}$$

b. $\dfrac{1}{x}-\dfrac{1}{x^2+x}$

<u>*Solution:*</u>

$$\frac{1}{x}-\frac{1}{x^2+x}=\frac{1}{x}-\frac{1}{x(x+1)}$$
$$=\frac{x+1}{x(x+1)}-\frac{1}{x(x+1)}$$
$$=\frac{x+1-1}{x(x+1)}$$
$$=\frac{x}{x(x+1)}$$
$$=\frac{1}{x+1}$$

c. $$\frac{xy^{-1}+yx^{-1}}{x^2+y^2}$$

Solution:

$$\begin{aligned}\frac{xy^{-1}+yx^{-1}}{x^2+y^2} &= \frac{\frac{x}{y}+\frac{y}{x}}{x^2+y^2}\\ &= \frac{\frac{x^2+y^2}{xy}}{x^2+y^2}\\ &= \frac{1}{xy}\end{aligned}$$

d. $$\frac{\sqrt{2-x^2}+x^2\left(2-x^2\right)^{-\frac{1}{2}}}{2-x^2}$$

Solution:

$$\begin{aligned}\frac{\sqrt{2-x^2}+x^2\left(2-x^2\right)^{-\frac{1}{2}}}{2-x^2} &= \frac{\left(2-x^2\right)^{-\frac{1}{2}}\left[\left(2-x^2\right)^{1}+x^2\right]}{2-x^2}\\ &= \frac{2-x^2+x^2}{\left(2-x^2\right)^{\frac{1}{2}}\left(2-x^2\right)^{1}}\\ &= \frac{2}{\left(2-x^2\right)^{\frac{3}{2}}}\end{aligned}$$

2. Simplify each expression:

a. $$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

b. $\dfrac{p^{-1}-q^{-1}}{(pq)^{-1}}$

c. $\dfrac{4}{x^2\sqrt{x^2+4}}+\dfrac{1}{\sqrt{x^2+4}+x}\cdot\left(\dfrac{x}{\sqrt{x^2+4}}+1\right)$

C. SOLVING EQUATIONS

Example 3: Solve each equation

a. $\frac{3}{2x-4}-\frac{5}{x+3}=\frac{2}{x-2}$

Solution:

$$\frac{3}{2x-4}-\frac{5}{x+3}=\frac{2}{x-2}\ ,\ \text{Note}: x\neq 2,-3$$

$$\frac{3}{2(x-2)}-\frac{5}{x+3}=\frac{2}{x-2}\ ,\ LCD=2(x-2)(x+3)$$

$$\frac{2(x-2)(x+3)}{1}\left[\frac{3}{2(x-2)}-\frac{5}{x+3}\right]=\frac{2(x-2)(x+3)}{1}\left[\frac{2}{x-2}\right]$$

$$3(x+3)-10(x-2)=4(x+3)\ ,\ \text{Linear}$$

$$3x+9-10x+20=4x+12$$

$$17=11x$$

$$x=\frac{17}{11}$$

b. $6x^2=3-7x$

Solution:

$$6x^2=3-7x\ ,\ \text{Quadratic}$$

$$6x^2+7x-3=0$$

$$(3x-1)(2x+3)=0$$

$$3x-1=0\ ,\ or\ \ 2x+3=0$$

$$x=\frac{1}{3},-\frac{3}{2}$$

c. $\dfrac{x}{6\sqrt{x^2+9}} - \dfrac{1}{8} = 0$

Solution:

$$\frac{x}{6\sqrt{x^2+9}} - \frac{1}{8} = 0$$

$$\frac{x}{6\sqrt{x^2+9}} = \frac{1}{8}$$

$$8x = 6\sqrt{x^2+9} \text{ , cross-multiply}$$

$$(4x)^2 = \left(3\sqrt{x^2+9}\right)^2$$

$$16x^2 = 9\left(x^2+9\right)$$

$$16x^2 = 9x^2 + 81$$

$$7x^2 = 81$$

$$x = \pm\frac{9}{\sqrt{7}} \text{ , but } x = -\frac{9}{\sqrt{7}} \text{ does not satisfy the original equation, therefore, one solution}$$

$$x = \frac{9}{\sqrt{7}}$$

3. Solve each equation

a. $\dfrac{-5}{3x-9} + \dfrac{4}{x-3} = \dfrac{5}{6}$

b. $(x+3)^2 = 5$

c. $3+\sqrt{3x+1}=x$

d. $1-\dfrac{1}{2\sqrt{1-x}}=0$

D. INEQUALITIES AND ABSOLUTE VALUE

Solving Inequations: typically the solution is an interval (or combination of intervals) that can be shown on a number line.

Example 4: Solve the inequality

a. $3+2x>7x-5$

Solution:

$$3+2x>7x-5$$
$$-5x>-8$$
$$x<\frac{8}{5}$$
$$or,\ \left(-\infty,\frac{8}{5}\right)$$
$$(\text{open interval})$$

b. $2x-3\le x+4<3x-2$

Solution:

$$2x-3\le x+4<3x-2$$
$$\text{separate: } 2x-3\le x+4, \text{ but, } x+4<3x-2$$
$$x\le 7\ \cap\ x>3$$
$$3<x\le 7$$
$$or,\ (3,7]\ (\text{closed interval})$$

c. $x^3+3x^2>4x$

Solution:

$$x^3+3x^2>4x$$
$$x^3+3x^2-4x>0$$
$$x\left(x^2+3x-4\right)>0$$
$$x(x+4)(x-1)>0$$
$$3 \text{ zeros}/4 \text{ regions to "test"}$$
$$\text{show: } -4<x<0,\ or,\ x>1$$
$$\text{or, } (-4,0)\cup(1,\infty)$$

ABSOLUTE VALUE: $|\ \ |$ The absolute value of any number is always positive. So when solving the equation $|x|=3$, there are two solutions: $x=\pm 3$.

Solving Inequations with absolute value: two cases

case (i) – if $|x|<a$, then $-a<x<a$

case (ii) – if $|x|>a$, then $x>a$, or, $x<-a$

Example 5: Solve the inequality

a. $|x| < 3$

Solution:

(case (i)), then $-3 < x < 3$

or, $(-3, 3)$

b. $|x| > 3$

Solution:

(case (ii)), then $x > 3$, or, $x < -3$

or, $(-\infty, -3) \cup (3, \infty)$

c. $|3x + 5| \le 8$

Solution:

(case (i)) then $-8 \le 3x + 5 \le 8$

$-13 \le 3x \le 3$

$\frac{-13}{3} \le x \le 1$

or, $\left[\frac{-13}{3}, 1\right]$

4. Solve the inequality

a. $-5 \le 3 - 2x \le 9$

b. $x^3 + 3x < 4x^2$

c. $|x + 1| \ge 3$

E. CARTESIAN (XY) PLANE/STRAIGHT LINES

Three Forms of Equations of a Line:

Slope/Intercept Form: $y = mx + b$

Point/Slope Form: $y - y_1 = m(x - x_1)$

General Form: $ax + by + c = 0$

Example 6: Find the equation of a line that goes through $\left(\frac{1}{2}, -\frac{2}{3}\right)$ and is perpendicular to the line $4x - 8y = 1$.

Solution:

$$4x - 8y = 1,\ 8y = 4x - 1,\ y = \frac{1}{2}x - \frac{1}{8}$$

$$\text{so, } m = -2$$

$$y = mx + b$$

$$y = -2x + b,\ \left(\frac{1}{2}, -\frac{2}{3}\right)$$

$$-\frac{2}{3} = -2\left(\frac{1}{2}\right) + b$$

$$b = \frac{1}{3}$$

$$\text{So, answer: } y = -2x + \frac{1}{3},\ \text{or, } 6x + 3y - 1 = 0$$

5. Find the equation of a line that goes through $(-1, -2)$ and $(4, 3)$.

F. TRIGONOMETRY

Radian Measure: ratio of arc length, *s*, divided by radius, *r* of a circle.

$$\theta = \frac{s}{r},\ 1\text{radian}\left(s = r\right);\ \theta = \frac{r}{r} = 1\text{rad} \approx 57.3^\circ$$

for full circle, $s = c = 2\pi r:\ \theta = \dfrac{2\pi r}{r} = 2\pi$

that is, $2\pi \text{ rad} = 360^\circ$

conversion ratios: $\left(\dfrac{\pi}{180}\right) \rightarrow$ from degrees to radians.

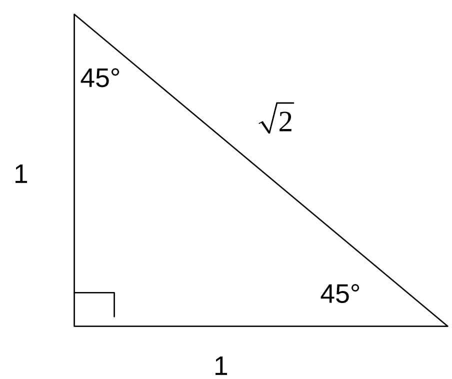
$\left(\dfrac{180}{\pi}\right) \rightarrow$ from radians to degrees.

Ex: $\dfrac{\pi}{4} : \dfrac{\pi}{4} \text{ rad}\left(\dfrac{180}{\pi}\right) = 45,\ \text{that is}: \dfrac{\pi}{4} = 45^\circ$

Some Special Angles:

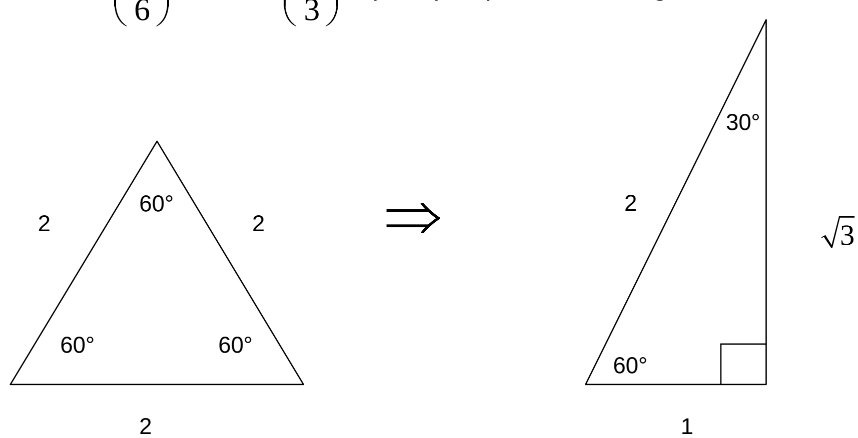
Isosceles Triangle:

For 45° $\left(\dfrac{\pi}{4}\right)$



$$\sin 45^\circ = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$
$$\cos 45^\circ = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$
$$\tan 45^\circ = \tan\frac{\pi}{4} = 1$$

For $30^\circ\left(\dfrac{\pi}{6}\right)$ and $60^\circ\left(\dfrac{\pi}{3}\right)$: (Slice) Equilateral Triangle



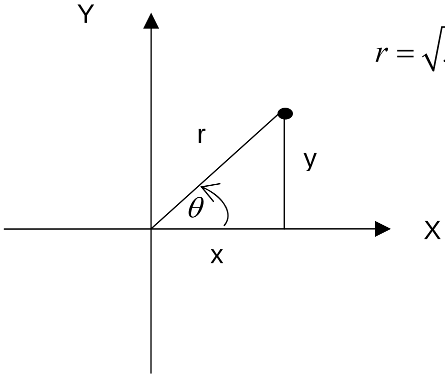
$$\sin 30^\circ = \sin\frac{\pi}{6} = \frac{1}{2}$$
$$\cos 30^\circ = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$
$$\tan 30^\circ = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$
$$\sin 60^\circ = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$
$$\cos 60^\circ = \cos\frac{\pi}{3} = \frac{1}{2}$$
$$\tan 60^\circ = \tan\frac{\pi}{3} = \sqrt{3}$$

Reciprocal Relations:

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Ex. $\sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Trigonometric Functions and Angles in the XY plane



$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}$$
$$\cos\theta = \frac{x}{r}$$
$$\tan\theta = \frac{y}{x}$$

(positive angles measured counter clockwise)

Example 7: find the exact value of sin 315°

Solution:

$$\sin 315^\circ = \sin\left(-45^\circ\right) = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$$

Note: The reference angle for 315° is 45°.

Trigonometric Identities (true for all Θ)

$\tan\theta = \frac{\sin\theta}{\cos\theta}$,

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta$,

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Example 8: Show $\cos(-\theta) = \cos\theta$

Solution:

$$\cos(-\theta) = \cos(0-\theta) = \cos 0 \cos\theta + \sin 0 \sin\theta$$
$$= 1 \cdot \cos\theta + 0 \cdot \sin\theta$$
$$= \cos\theta$$

Solving Trigonometric Equations (true for some Θ)

Since trigonometric functions are periodic, typically there are infinitely many solutions when solving these equations. To simplify the solution, an interval is usually chosen to be $0 \le \theta \le 2\pi$, or, $[0, 2\pi]$ (i.e., one full circle).

Even in one full circle each primary trigonometric function has two solutions due to the nature of their ratios in the 4 quadrants. To help find the second solution, we can apply the following **Properties**:

For $0 \le x \le 2\pi$, once x_1 is found, x_2 can be found as follows:

$$\text{for } \sin x : x_2 = \pi - x_1$$
$$\text{for } \cos x : x_2 = 2\pi - x_1$$
$$\text{for } \tan x : x_2 = \pi + x_1$$

Example 9: Solve $\cos x = \sin 2x$, in $[0, 2\pi]$ ***Solution:***

$$\cos x = \sin 2x$$
$$\cos x = 2\sin x \cos x$$
$$\cos x - 2\sin x \cos x = 0$$
$$\cos x(1 - 2\sin x) = 0$$

$\cos x = 0$ or $1 - 2\sin x = 0$

$$x_1 = \cos^{-1}(0)(\text{optional})$$
$$x_1 = \frac{\pi}{2}$$
$$\to x_2 = 2\pi - \frac{\pi}{2}$$
$$x_2 = \frac{3\pi}{2}$$

$$1 - 2\sin x = 0$$
$$\sin x = \frac{1}{2}$$
$$x_1 = \sin^{-1}\left(\frac{1}{2}\right)(\text{optional})$$
$$x_1 = \frac{\pi}{6}$$
$$\to x_2 = \pi - \frac{\pi}{6}$$
$$x_2 = \frac{5\pi}{6}$$

So, there are four solutions in $[0, 2\pi]$: $x = \frac{\pi}{6}, \ \frac{\pi}{2}, \ \frac{5\pi}{6}, \ \frac{3\pi}{2}$ (in order)

6. Find the remaining trigonometric ratios if $\csc x = \frac{-3}{2\sqrt{2}}$, $\pi < x < \frac{3\pi}{2}$

7. Find the exact value of $\sin 105^\circ$.

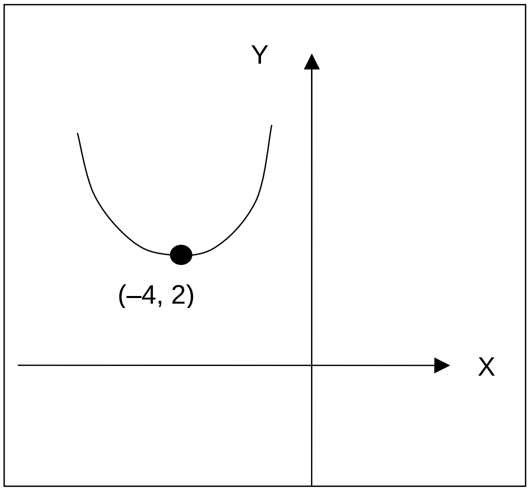
8. Find all values of x in the interval $[0, 2\pi]$ that satisfy the trigonometric equation: $2 + \cos 2x = 3\cos x$.

G. FUNCTIONS AND FUNCTION NOTATION

Example 10: Graph the function $y = x^2 + 8x + 18$ and state: domain, range, increasing vs decreasing, vertex, basic function and corresponding shifts.

Solution:

$$y = x^2 + 8x + 18$$
$$y = x^2 + 8x + \underline{16} - \underline{16} + 18$$
$$y = (x+4)^2 + 2, \text{ parabola with vertex} (-4, 2) (\text{opens up})$$



Domain, D: all x, or, $(-\infty, \infty)$

Range, R: $y \geq 2$, or, $[2, \infty)$

decreasing: $(-\infty, -4]$

increasing: $[-4, \infty)$

basic parabola:

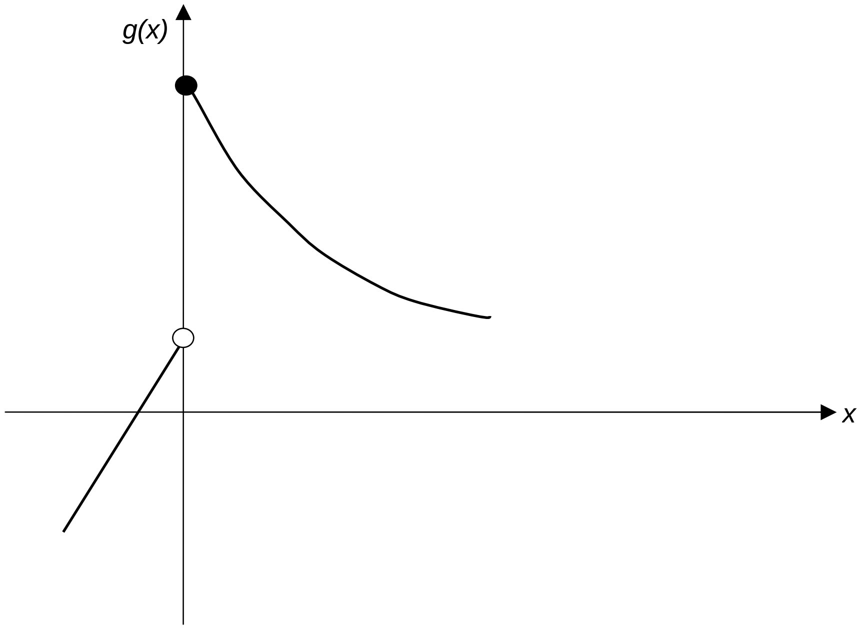
$$f(x) = x^2$$
$$\rightarrow y = \underbrace{f(x+4)}_{\text{horizontal shift}} \underbrace{+2}_{\text{vertical shift}}$$

Example 11: Sketch a graph of the piecewise function $g(x) = \begin{cases} 2x+1, \text{ if } x < 0 \\ -\sqrt{x} + 3, \text{ if } x \geq 0 \end{cases}$ and find:

a. $g(-2)$

b. $g(0)$

c. x, such that $g(x) = -1$



a. $g(-2) = 2(-2) + 1 = -3$

b. $g(0) = -\sqrt{0} + 3 = 3$

c.
$$-1 = 2x + 1 \quad \text{, or, } \quad -1 = -\sqrt{x} + 3$$
$$2x = -2 \qquad\qquad \sqrt{x} = 4$$
$$x = -1 \quad \text{, or, } \quad x = 16$$

$\underbrace{\text{Combinations}}_{+,-,\times,\div}$ and $\underbrace{\text{Composition}}_{\substack{f(g(x)) = f \circ g \\ g(f(x)) = g \circ f}}$ of functions.

Example 12: Given $f(x) = \sqrt{x-5}$ and $g(x) = x^3$, find:

a. $(f+g)x$

b. $(g/f)x$

c. $f \circ g$

d. $g \circ f$

e. $g \circ g$

Solutions:

a. $(f+g)x = f(x)+g(x)$
$= \sqrt{x-5}+x^3$

b. $(g/f)x = \dfrac{g(x)}{f(x)} = \dfrac{x^3}{\sqrt{x-5}},\ x \neq 5$

c. $f \circ g = f(g(x)) = \sqrt{x^3-5}$

d. $g \circ f = g(f(x)) = \left(\sqrt{x-5}\right)^3$
$= (x-5)^{\frac{3}{2}}$

e. $g \circ g = g(g(x)) = \left(x^3\right)^3 = x^9$

9. Find the domain and range of $y = \sqrt{x-2}-3$. Sketch a graph.

10. Find $f \circ g$, if $f(x) = \dfrac{1}{1-x}$ and $g(x) = \dfrac{1}{x}$.

11. Express $F(x)$ in the form $f \circ g$. In other words, identify $f(x)$ and $g(x)$. $F(x) = \sin\sqrt{x}$

Answer Key For MATH 100 Review Package

1. a. $\frac{-13}{9}$ b. $\frac{-3}{2}$ c. $\sqrt{7}$

2. a. $\frac{-1}{x(x+h)}$ b. $q-p$ c. $\frac{\sqrt{x^2+4}}{x^2}$

3. a. $\frac{29}{5}$ b. $-3\pm\sqrt{5}$ c. 8 d. $\frac{3}{4}$

4. a. $[-3, 4]$ b. $(-\infty, 0)\cup(1, 3)$ c. $(-\infty, -4]\cup[2, \infty)$

5. $y = x-1$

6. $\sin x = -\frac{2\sqrt{2}}{3}$

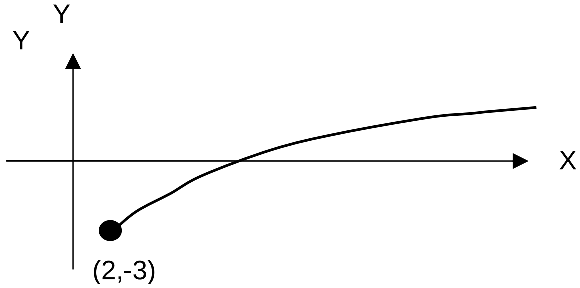
$\cos x = -\frac{1}{3}$, $\sec x = -3$

$\tan x = 2\sqrt{2}$, $\cot x = \frac{1}{2\sqrt{2}}$

7. $\frac{\sqrt{3}+1}{2\sqrt{2}}$

8. $x = 0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

9. Y



domain, D: $[2, \infty)$

range, R: $[-3, \infty)$

10. $\frac{x}{x-1}$

11. $f(x) = \sin x,\ \ g(x) = \sqrt{x}$

$f\circ g = f(g(x)) = \sin\sqrt{x} = F(x)$

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